Art. VIII.—*Remarks on some New Tables for Finding Heights by the Barometer.*

By E. J. White, F.R.A.S.

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Since the memorable 19th of September, 1648, when the celebrated Pascal ascended the Puy de Dome, in the French Province of Auvergne, and found, as he had anticipated, that the mercury in his barometer fell as he rose, the barometric method of measuring heights has been extensively used, and in many instances, it is the only one available. Until about thirty years ago, only mercurial barometers were used for this purpose. They are still the most accurate, and should always be employed where the utmost precision of this method of measuring is required, but they have the disadvantage of being expensive, bulky, and fragile. At the period mentioned above, the aneroid, which had been invented in 1850 by M. Vidi, came into general use, and of late years it has become so popular, that as a weather-glass, or tourist's companion for measuring elevations, it has nearly superseded the tube of mercury. Almost the sole reason for this preference, is its portability; good aneroids are made that will go into the waistcoat pocket, while others are small enough to be used as charms to be suspended from the watch chain; these latter, however, are to be considered more as trinkets than as philosophical instruments. On the other hand, it is not an independent instrument, but has to have its scale originally marked off, and its errors from time to time found, from comparison with the mercurial barometer, than which it is also more complicated and less stable.

The mathematical part of the subject has been treated by many eminent writers; but since the time of Laplace, his formula has formed the basis of the investigations. If air was an incompressible fluid like water, the law would be very simple; the pressures would be proportional to the heights, and if its density was throughout the same as at the surface of the earth, it would be all contained within a limit of about five miles. Air, however, is an elastic fluid, subject to Mariotte's law, and has therefore its density proportional to the pressure to which it is subjected, and if there were no disturbing elements, the law would still be a simple one. The heights would be in arithmetical progression, while the pressures would be in geometrical progression, thus having the nature of a table of logarithms, and the difference in height would be equal to a certain constant distance, multiplied by the difference between the logarithms of the heights of the barometer at the two places. The principal disturbing element is temperature, which varies the density of the air by its changes. If we could obtain the temperature of the column of air between the two places, the proper correction could be obtained; but as we can only apply the thermometer to the air near the surface of the earth, where it is greatly affected by radiation, and have to assume, as Laplace has done, that the temperature of the column of air between the two stations is the mean of that near the ground at each station, we sometimes get very anomalous results. This has been proved by observing barometers at two stations, not very distant, though considerably differing in altitude, whose difference of height had been well determined by careful levelling. Having now the height and the barometer readings, we can substitute them in Laplace's formula, and work out the temperatures. An extensive series of such observations was made by Professor Plantamour, using for his stations, Geneva and the Hospice of St. Bernard. From the mean of the 8 a.m. observations, the correction to the observed mean temperature varied from $+4{\cdot}5°$ in December to $-4{\cdot}3°$ in July. The 4 p.m. observations gave $+2{\cdot}3°$ in December, and $-6{\cdot}5°$ in July, and the 10 p.m. observations had a range of from $+4{\cdot}7°$ in December to $-0{\cdot}2°$ in July. Similar observations in the United States, between Mount Washington in New Hampshire and Portland, Maine, gave the following ranges: 8 a.m., $+1{\cdot}7°$ in August, and $-1{\cdot}9°$ in November; 5 p.m., $-3{\cdot}2°$ in December, and $-1{\cdot}1°$ in September; and 11 p.m., $+3{\cdot}4°$ in August, and $-2{\cdot}1°$ in November. As the effect of an error of one degree in the mean temperature is about 2 feet in 1000, the greatest of the above corrections, the $-6{\cdot}5°$ in July, would amount to 13 feet in 1000. Attempts have been made to determine the law of decrement of the temperature of the atmosphere, the results are not very consistent, but it appears to be nearly in an arithmetical progression, Glaisher finding in his 20,000 feet balloon ascent, a decrease of 1° for each 300 feet of ascent. The mean of several observations in Europe and America, is 1° for every 308 feet.

Humidity is another disturbing element: aqueous vapour has only five-eighths of the density of dry air, and does not permanently retain the gaseous form; the hygrometer is, therefore, sometimes used in conjunction with the barometer and thermometer in hypsometry, but as the necessary correction is not well established, it is generally neglected.

The last important source of error is that of gradient. With the air in a state of static equilibrium, two stations at the same height would receive equal pressures; but as a matter of fact, we find that stations at the same level have at the same moment very different pressures, especially if they are distant from one another. Gradients are the result of such complicated conditions, that there is no way of eliminating their effects. We can only mitigate them by taking extensive series of observations, on the supposition, that the effects will thus balance one another in the mean result.

Of the many forms of mercurial barometers, I think the Gay Lussac syphon is best suited for mountain work. It is very compact, not liable to derangement, and eliminates the error of capillarity. The zero point is now usually placed in the middle, so that the sum of the upper and lower readings gives the height of the quicksilver. Of aneroids, the Watkin seems likely to become the favourite for professional work, owing to its very open scale, which, in a four inch instrument, has about seven inches of its outer divisions to denote a variation of one inch of mercury. Ordinary travellers, however, will generally avail themselves of the small instruments, of from about one and a half to two inches diameter, which can be easily carried in the waistcoat pocket, where they are generally much less liable to injury than the larger ones carried on a leather strap. It must be mentioned, however, that no system of barometric measuring will give results nearly approaching those obtained by levelling, or even by the measurement of the vertical angles, where the uncertainty of the refraction is a disturbing element.

From a work issued in 1882 by the Geological Survey Department of the United States, entitled "A New

Method of Measuring Heights by means of the Barometer," by G. K. Gilbert, the following table is given, from the investigation of actual measurements. The assumptions are that one station is 5000 feet higher than the other, that the two places are fifty miles apart, and that they are situated in the temperate zone, remote from the ocean. The observations are supposed to be made near the middle of a fine summer's day, with a light wind blowing:—

——	Probable error in feet.	Possible error in feet.
From annual gradient	6	20
,, Diurnal gradient	8	30
,, Non-periodic gradient	20	50
,, Temperature	100	300
,, Moisture	10	20
,, Imperfection of observation ..	10	Unlimited
Totals	103	420 +

It will be noted how largely the temperature error exceeds all the others; and from the second column it will be seen that, excluding the error of observation (which, owing to mistakes, may be unlimited) the total error may amount to 420 feet, or more than one-twelfth of the whole amount measured. The probable error of 10 feet assigned for imperfection of observation is meant for a mercurial barometer; for an aneroid it would be much larger, even when used with the greatest care, and frequently compared with the mercurial column for finding its errors. At our observatory, we have an apparatus for testing mountain and other aneroids. I append the results of some of the late comparisons:—No. 1 has an error of ·09 in a range of 26 to 30 inches. No. 2, ·30 in a range from 23 to 30 inches. No. 3, ·13, range 24 to 30; and ·18, range 23 to 30. No. 4, ·13, range 24 to 30; ·23, range 23 to 30. No. 5 (which is a French barometer metrically divided), ·40, range 25 to 30; and ·63, 24 to 30. In the last three instruments it will be remarked, how the error increases as the extremity of its scale is reached. I have on several occasions, when travelling on our railways, carried an aneroid with me, and recorded its readings, as well as the temperature of the air at stations where we have stopped, and have compared the heights thus deduced with the accurate ones of the railway levels. At first I treated the results as matters of curiosity only, and did not preserve the records, but I give those I have been able to find. On March 21, 1883, I measured the heights of the following stations on the Sandhurst line, with the French barometer mentioned above. I left Melbourne at 6.45 a.m., the barometer reading 775 millimetres, and temperature of the air 20° centigrade. When I reached Kangaroo Flat at 10.46 a.m. the barometer read 748 millimetres, and the temperature of the air was 24·6° centigrade. The following are the corrections in feet to be applied to the aneriod heights to get the true heights: Sunbury + 26, Lancefield Junction + 105, Gisborne + 50, Macedon − 9, Woodend + 96, Malmsbury − 48, Taradale + 12, Castlemaine + 34, Kangaroo Flat − 34. On May 8 of the present year, I took No. 3 of the above-mentioned barometers along part of the same line. I started at 7 a.m., the barometer reading 30·30 in., and the temperature of the air being 58° Fahr. I reached Macedon at 8.45; the barometer reading 28·73, and the temperature of the air 66°. I left Macedon at 5.30 p.m.; barometer 28·65, air 66°, and arrived in Melbourne about 7 p.m.; barometer 30·28, air 62°. Most of my levelling this day was done off the railway, where I had no levels for comparison, but the known heights measured required the following corrections: Gisborne + 154, Macedon (from up readings) + 151, and from the return readings + 86. The above results are not given as specimens of the best method of using the aneroid, but the observations were made with the greatest care. When the mercurial barometer is used, and the stations are in the same vertical, where the error of gradient should be very little, the errors are generally greater than expected. One of the most extensive series of this description was carried out from July 22 to October 15, 1848, by the Royal Engineers employed in the trigonometrical survey of Great Britain. Four mercurial barometers were used, two of these were placed on the top of the dome of St. Paul's Cathedral in London, the other two rested on the floor under the centre of the dome; after 62 observations had been taken the barometers were transposed, and 89 additional observations were made. The mean of the first set made the height 356·99 feet, and the mean of the second set 353·57, whereas the actual height was 352·75; both determinations were therefore in excess of the real amount—the first to the extent of 4·24 feet, and the second to 0·82 feet. Most mountain aneroids are furnished with an altitude scale, either on the same dial as the inch divisions or on a separate ring which can be set to zero at starting, and thus save the trouble of taking the differences. This latter plan, although more convenient, is not so accurate as the former; but in every case, if the best result is desired, the readings should be taken from the scale of inches or millimetres, as it is uncertain to what temperature the altitude scale is adapted. I have measured some of the English ones, and find the temperatures they represent to vary from 47° to 51°, but the French one mentioned before has its scale adapted to 70° Fahr. The small tables supplied by instrument makers, which have no temperature factor, are generally adapted to about 50°; while the larger tables, which contain the factors for temperature, latitude, and decrease of gravity, are usually given for 32°, the freezing point. The adoption of so low a temperature as 32° is very inconvenient for Australia, where the mean temperature is much higher, as it necessitates a very large correction, and as this correction is frequently neglected, the results must be very inaccurate. I have therefore thought it desirable to compute a new table, in which the mean temperature and middle latitude of this part of Australia should be used. Calling B the height of the barometer, and t the temperature of the air at the lower station, B′ and t′ the same quantities for the upper station, L the latitude, and A the difference of height in feet between the two stations. Laplace's formula, leaving out the factors depending on the decrease of gravity, may be written:

$$A = 60158{\cdot}6\ (\log. B - \log. B') \times \left(\frac{1 + t + t' - 64}{900}\right) 0{\cdot}00265 \cos 2\,L.$$

Taking now T and T′ equal to 60°, and L = 37°, and substituting them in the above equation, we get

$$A = 63948{\cdot}6\ (\log. B - \log. B').$$

The mean height of the barometer at the Melbourne observatory, 91 feet above the level of the sea, reduced to 32° Fahr., is 29·931 in.; increasing this by 0·07 to bring it up to 60° Fahr., nearly the mean temperature of Melbourne, and by 0·10 to reduce it to sea level, we get 30·10; with this value in the above equation we should find 63948·6 log. B = 94552·26. It would be more convenient, however, to have it represented by o, because the tabulated values corresponding to the height of the barometer will at once show the height above the level of the sea in Melbourne in the normal state of the atmosphere. This arrangement will have the disadvantage of introducing negative numbers, which will represent depths below the level of the sea, and the quantities will diminish with an increase of the argument, but on the whole, I consider it better than the usual plan. Let now F = 94552·26 – 63948·6 B, and F′ = 94552·56 – 63948·6 B′, and calling *a* the height between the two places at the mean adopted temperature, 60°, we have a = F′ – F. The values of F or F′ are tabulated in the first of the present tables, with the height of the barometer for argument. If a mercurial barometer has been used, the above value of a should be corrected for the difference between the temperatures of the quicksilver at the two stations. If T and T′ represent these temperatures at the lower and upper stations respectively, the correction will be 2·5 (T′ – T); as T′ is nearly always less than T, this correction will be generally negative.

It will now be necessary to correct *a* for the actual temperature of the air, calling this corrected value A, we have—

$$A = a + \frac{a}{956{\cdot}7}\,(t + t' - 120)$$

putting $n = 1 + \frac{t+t'-120}{956{\cdot}7}$ we have $A = na$; and the value of n is given in the second table with t + t′ as argument. Generally speaking, the value determined from the equation $A = n\,(F' - F)$ will be far within the limits of accuracy of the aneroid barometer; but if it should be thought desirable to apply the small corrections depending on latitude, and decrease of gravity on the vertical, small tables are given for the purpose.

TABLE I.

Argument—Height of Barometer in Inches.

B.	F.	Diff.	B.	F.	Diff.	B.	F.	Diff.
23·0	7471·7	120·5	26·0	4066·7	106·6	29·0	1034·0	95·6
·1	7351·2	120·0	·1	3960·1	106·2	·1	938·4	95·3
·2	7231·2	119·5	·2	3853·9	105·8	·2	843·1	95·0
·3	7111·7	118·9	·3	3748·1	105·4	·3	748·1	94·6
·4	6992·8	118·4	·4	3642·7	105·0	·4	653·5	94.3
·5	6874·4	117·9	·5	3537·7	104·6	·5	559·2	94·0
·6	6756·5	117·5	·6	3433·1	104·2	·6	465·2	93·7
·7	6639·0	116·9	·7	3328·9	103·8	·7	371·5	93·3
·8	6522·1	116·5	·8	3225·1	103·5	·8	278·2	93·1
·9	6405·6	115·9	·9	3121·6	103·0	·9	185·1	92·7
24·0	6289·7	115·5	27·0	3018·6	102·7	30·0	+ 92·4	92·4
·1	6174·2	115·0	·1	2915·9	102·3	·1	0·0	92·1
·2	6059·2	114·5	·2	2813·6	101·9	·2	− 92·1	91·8
·3	5944·7	114·1	·3	2711·7	101·6	·3	− 183·9	91·5
·4	5830·6	113·6	·4	2610·1	101·1	·4	− 275·4	91·2
·5	5717·0	113·1	·5	2509·0	100·8	·5	− 366·6	91·0
·6	5603·9	112·7	·6	2408·2	100·5	·6	− 457·6	90·6
·7	5491·2	112·2	·7	2307·7	100·1	·7	− 548·2	90·3
·8	5379·0	111·7	·8	2207·6	99·7	·8	− 638·5	90·0
·9	5267·3	111·3	·9	2107·9	99·4	·9	− 728·5	89·7
25·0	5156·0	110·9	28·0	2008·5	99·0	31·0	− 818·2	89·5
·1	5045·1	110·4	·1	1909·5	98·6	·1	− 907·7	89·1
·2	4934·7	110·0	·2	1810·9	98·3	·2	− 996·8	88·9
·3	4824·7	109·6	·3	1712·6	98·0	·3	−1085·7	88·6
·4	4715·1	109·1	·4	1614·6	97·6	·4	−1174·3	88·3
·5	4606·0	108·7	·5	1517·0	97·3	·5	−1262·6	88·0
·6	4497·3	108·3	·6	1419·7	96·9	·6	−1350·6	87·8
·7	4389·0	107·8	·7	1322·8	96·6	·7	−1438·4	87·4
·8	4281·2	107·5	·8	1226·2	96·3	·8	−1525·8	87·3
·9	4173·7	107·0	·9	1129·9	95·9	·9	−1613·1	86·9
26·0	4066·7		29·0	1034·0		32·0	−1700·0	

TABLE II.

Argument—Sum of Temperatures of Air at both Stations.

$t+t'$	n	$t+t'$	n	$t+t'$	n	$t+t'$	n	$t+t'$	n
50	0·9268	80	0·9582	110	0·9895	140	1·0209	170	1·0522
51	·9279	81	·9592	111	·9906	141	1·0220	171	1·0533
52	·9289	82	·9603	112	·9916	142	1·0230	172	1·0543
53	·9300	83	·9613	113	·9927	143	1·0240	173	1·0553
54	·9310	84	·9624	114	·9937	144	1·0251	174	1·0564
55	·9321	85	·9634	115	·9948	145	1·0261	175	1·0575
56	·9331	86	·9645	116	·9958	146	1·0272	176	1·0585
57	·9341	87	·9655	117	·9969	147	1·0282	177	1·0596
58	·9352	88	·9666	118	·9979	148	1·0293	178	1·0606
59	·9362	89	·9676	119	0·9990	149	1·0303	179	1·0617
60	·9373	90	·9686	120	1·0000	150	1·0314	180	1·0627
61	·9383	91	·9697	121	1·0010	151	1·0324	181	1·0638
62	·9394	92	·9707	122	1·0021	152	1·0334	182	1·0648
63	·9404	93	·9718	123	1·0031	153	1·0345	183	1·0659
64	·9415	94	·9728	124	1·0042	154	1·0355	184	1·0669
65	·9425	95	·9739	125	1·0052	155	1·0366	185	1·0679
66	·9436	96	·9749	126	1·0063	156	1·0376	186	1·0690
67	·9447	97	·9760	127	1·0073	157	1·0387	187	1·0700
68	·9457	98	·9770	128	1·0084	158	1·0397	188	1·0711
69	·9467	99	·9780	129	1·0094	159	1·0408	189	1·0721
70	·9477	100	·9791	130	1·0105	160	1·0418	190	1·0732
71	·9488	101	·9801	131	1·0115	161	1·0429	191	1·0742
72	·9498	102	·9812	132	1·0125	162	1·0439	192	1·0753
73	·9509	103	·9822	133	1·0136	163	1·0449	193	1·0763
74	·9519	104	·9833	134	1·0146	164	1·0460	194	1·0773
75	·9530	105	·9843	135	1·0157	165	1·0470	195	1·0784
76	·9540	106	·9854	136	1·0167	166	1·0481	196	1·0794
77	·9551	107	·9864	137	1·0178	167	1·0491	197	1·0804
78	·9561	108	·9875	138	1·0188	168	1·0502	198	1·0815
79	·9571	109	·9885	139	1·0199	169	1·0512	199	1·0826
80	·9582	110	·9895	140	1·0209	170	1·0522	200	1·0836

Small Corrections, depending on Decrease of Gravity at the Upper Station, Height of the Barometer at the Lower Station, and the Latitude.

		A						
		1000	2000	3000	4000	5000	6000	7000
Decrease of Gravity ..		+2·5	5·2	7·4	10·8	13·7	16·7	19·9
	B.							
Height of Barometer at Lower Station ..	24°	+0·6	1·2	1·8	2·4	3·0	3·6	4·2
	25°	+0·5	1·0	1·5	2·0	2·8	2·9	3·4
	26°	+0·4	0·8	1·2	1·6	2·0	2·3	2·7
	27°	+0·3	0·6	0·9	1·2	1·4	1·7	2·0
	28°	+0·2	0·4	0·6	0·8	0·9	1·1	1·3
	L.							
Latitude ..	20°	+1·3	2·6	3·9	5·2	6·5	7·8	9·1
	25°	+1·0	1·9	2·9	3·9	4·9	5·8	6·8
	30°	+0·6	1·2	1·8	2·4	3·0	3·6	4·2
	35°	+0·2	0·3	0·5	0·7	0·9	1·1	1·2
	40°	−0·3	−0·5	−0·8	−1·1	−1·3	−1·6	−1·9
	45°	−0·7	−1·5	−2·2	−2·9	−3·6	−4·4	−5·1

$$A = n\,(F' - F) + \text{small corrections.}$$

Rule.—Take out the values of F corresponding to the heights of the barometer at the upper and lower stations; take their difference, which in case one of them is negative will be their numerical sum, multiply the difference by the value of n corresponding to the sum of the temperatures of the air at each station; the product A will be very nearly the height in feet of one station above the other. To this value, apply the small corrections for a final result. If the barometer be a mercurial one, a further correction of 2½ feet will have to be subtracted for each degree of excess of the temperature of the mercury at the lower station over that of the upper.

Example.—On May 8, 1889, an aneroid barometer, at the meteorological station near the summit of Mount Macedon, indicated 27·03 in., the temperature of the air being 59°; the same barometer, a few hours after, read at the Melbourne Railway Station 30·23 in., the temperature of the air being 62°. Find the difference of height:—

27·03 in. F =	2987·8
30·23 in. =	−119·6
Diff.	3107·4
	100·1 = n for 121°
	3107·4
	3·1
	3110·5
Small corrections +	8·2
Difference of height ..	3118·7